



# Numerical Simulation of Vehicles Queue Induced by Traffic Light Signals Based on Macroscopic Approach

Dede Tarwidi\*, Erwin Budi Setiawan, and Rian Febrian Umbara

School of Computing, Telkom University

Jalan Telekomunikasi No. 1 Terusan Buah Batu, Bandung 40257, Indonesia

\*Corresponding author E-mail: [dedetarwidi@telkomuniversity.ac.id](mailto:dedetarwidi@telkomuniversity.ac.id)

## Abstract

A disproportionate adjustment of traffic light signals on crowded intersections has become one of the cause of a traffic congestion in big cities. To control traffic congestion in a signalized intersection, it is required a prediction of vehicles queue length induced by traffic light signals every time. In this paper, numerical simulation of vehicles queue in three roads which is connected by an intersection is presented. The Lighthill–Whitham–Richards (LWR) model which is considered as macroscopic model is used to describe traffic density at the single road. Godunov method is adopted to obtain numerical solution of LWR model. The numerical results of vehicles queue length are then compared by observational data at a signalized intersection in Bandung city. The LWR model can predict the vehicles queue length with the accuracy is approximately 85.33%.

**Keywords:** LWR model, Godunov method, traffic congestion, signalized intersection, numerical simulation..

## 1. Introduction

The number of vehicles in cities with large population such as Jakarta and Bandung, day by day, is continuously increasing. The high number of vehicles growth has not been balanced yet with the road infrastructures and the appropriate government policies.

This made traffic congestion in big cities is getting worse. One of the cause of heavy traffic congestion is disproportionate coordination of traffic light signals. In the real situation, the traffic light signals may not be coordinated with the number of arrival and leaving of vehicles. Ideally, the period of traffic light signals can adjust automatically with the length of vehicles queue. It is called adaptive traffic light system. However, in fact, in some signalized intersections, the traffic light signal is manually controlled by policeman adjusting with the traffic condition.

Many researchers have studied traffic the congestion induced by traffic light signals via modeling and numerical simulation. Some of them focused on optimization of traffic light system. Lo (1999) developed traffic signal control with mixed integer programming technique. He showed that his numerical results for timing plan is better than conventional queue management practice. Lo, Chang, & Chan (2001) used cell transmission model (CTM) to develop dynamics traffic control system. Their system is designated as dynamics intersection signal control optimization (DISCO). Mirchandani & Head (2001) discussed real-time traffic-adaptive signal control system. Their simulation results showed good agreement with the Laboratory test results. Lin & Wang (2004) also discussed traffic signal optimization problem by using enhanced 0-1 mixed-integer programming. Their model was based on cell transmission model which is equipped by number of stops, dynamics cycle length, and lost time. Moreover, Gradinescu et al. (2007) presented adaptive traffic light system. They concluded that the traffic fluency in intersections can be significantly improved. Yuan, Zhao, & An (2014) used ant colony algorithm to optimize signal timing plan on bottlenecks. They concluded that their method can mitigate traffic congestion at intersection by tuned the signal timing. Arara, Abousetta, & Drebbe (2015) developed an algorithm to dynamically reallocate traffic light time slots based on traffic density on an intersection. Their system showed that multi-junction traffic control can be handled to reduce traffic congestion.

Vehicles queue length has become an important factor in building traffic control system so that other researchers studied a model than can predict queue vehicles length. Yang & Yagar (1995) modeled traffic assignment and optimized signal timing in saturated road networks. Their model can predict queue length in saturated networks. Wong & Wong (2002) used enhanced LWR model to develop multi-class traffic flow model with heterogeneous drivers. Their numerical results showed that the model can predict emerging traffic flow phenomena. Van Leeuwen (2006) discussed fixed-cycle traffic-light (FTCL) queue. The numerical procedure is also proposed to obtain the performance characteristics. Moreover, Wen (2008) proposed a framework for a dynamic and automatic traffic light control expert system. His system can simulate the arrival and leaving number of cars on roads. Liu et al. (2009) estimated queue length of vehicles in real-time at intersection with traffic light signal. They applied Lighthill–Whitham–Richards (LWR) model to the system and showed that their approach can estimate long queues with satisfactory accuracy.

In this work, numerical simulation of vehicles queue length induced by traffic light signals is studied. The LWR model which is used to describe traffic density in a single road is presented in Section 2. The LWR model can be considered as macroscopic model, i.e. the vehi-

cles dynamics is treated like fluid motion. Numerical solution of the LWR equation is discussed in Section 3. In Section 4, the length of vehicles queue at a signalized intersection is discussed. The numerical results are compared with the observational data.

## 2. Traffic Flow Modeling

Traffic flow on single road can be simply represented by LWR model. After Lighthill & Whitham (1955), and followed by Richards (1956) proposed a model for traffic flow. This model is based on conservation of vehicles in a road section which is called continuity equation. The LWR model requires relation between vehicles speed and density of vehicles. Suppose  $\rho(x, t)$  is density of vehicles, that is the number of vehicles per unit length. The continuity equation is given by (Haberman, 2009; Guinot, 2003; Treiber & Kesting, 2013;)

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial t} = 0 \quad (1)$$

where  $q(x, t)$  is the number of vehicles through a point per unit time. The  $q(x, t)$  is also called vehicles flux. Note that  $\rho \in [0, \rho_{\max}]$  and  $\rho_{\max}$  is the maximum density of vehicles. The  $\rho_{\max}$  occurs when the speed of vehicles is zero. Moreover, the vehicles flux  $q(x, t)$  is assumed as function of vehicles speed. It can be written as

$$q = \rho v \quad (2)$$

where  $v$  is the average speed of vehicles.

It is assumed that the vehicles speed is changed instantaneously depending on traffic condition. If the traffic density is increasing then the vehicles speed is adapting to slow down. As consequence,  $v$  is a function of  $\rho$  only. The speed of vehicles is given by (Guinot, 2003)

$$v = \left(1 - \frac{\rho}{\rho_{\max}}\right) v_{\max} \quad (3)$$

where  $v_{\max}$  is maximum speed of vehicles. It occurs when the density of vehicles is approaching zero. Then, the vehicles flux can be written as

$$q = \left(1 - \frac{\rho}{\rho_{\max}}\right) v_{\max} \rho \quad (4)$$

## 3. Numerical Solution

Equation can be solved by using Godunov method. The first step of Godunov method is discretization of space and time. The technique of discretization is based on finite volume method. Let  $l$  is the length of road and  $t_f$  is the final time of simulation. Suppose  $M$  is the number of cell and  $N$  is the number time level. Then,  $\Delta x = l/M$  and  $\Delta t = t_f/N$  are space and time step, respectively. Let  $V_i = [x_{i-1/2}, x_{i+1/2}]$  denotes cell  $i$  and  $x_i$  is a middle point of  $V_i$ . Here, the subscript  $i + 1/2$  represents interface between cell  $i$  and cell  $i+1$ . More comprehensive discussion about Godunov method, the reader can read literatures (Tarwidi & Pudjaprasetya, 2013; Tarwidi, 2015).

Suppose  $\rho_i^n = \rho(x_i, t_n)$  be average density in cell  $i$  at time level  $n$ . The density of vehicles according to Godunov method is given by

$$\rho_i^{n+1} = \rho_i^n + \frac{\Delta t}{\Delta x} (q_{i-1/2}^{n+1/2} - q_{i+1/2}^{n+1/2}) \quad (5)$$

where  $q_{i-1/2}^{n+1/2} = q(x_i, t^{n+1/2})$  is vehicles flux at point  $x_i$  and time  $n + 1/2$ . The superscript  $n + 1/2$  denotes average time between time level  $n$  and  $n + 1$ . The flux  $q_{i-1/2}^{n+1/2}$  can be calculated as

$$q_{i-1/2}^{n+1/2} = \left(1 - \frac{\rho_{i-1/2}^{n+1/2}}{\rho_{\max}}\right) v_{\max} \rho_{i-1/2}^{n+1/2} \quad (6)$$

where  $\rho_{i-1/2}^{n+1/2}$  is the solution of Riemann problem defined as:

The solution of Riemann problem (7) is discussed comprehensively in (LeVeque, 2002; Guinot, 2003; Toro, 2013).

## 4. Results and Discussion

In this section, the vehicles queues length induced by traffic light signals at a intersection is simulated and discussed. The simulation results are compared to observational data which was taken at a signalized intersection. The intersection is located in Terusan Buah Batu street and in front of Buah Batu Toll Gate, Bandung City. The observation of traffic flow is conducted by taking videos from the sky

using a drone. From these videos, duration of the red and green lights and vehicles queue length on every road are calculated. Fig.1 shows a video snapshot of vehicles queues in the intersection which was taken by a drone. From the figure, it can be seen that there are three sections of roads that will be observed, i.e. Bojongsoang – Buah Batu, Buah Batu – Bojongsoang, and Exit Toll Gate of Buah Batu.

Since the LWR model is valid for single road, then in this study, the vehicles queues data were taken only for single lane from each road section being observed. The length of vehicles queue is calculated by counting the number of vehicles which have zero speed (stopped state). For the simplicity, the presence of motorcycles is ignored, so the calculated vehicles is only cars. The duration of traffic light signals and vehicles queues length based on the observation are summarized in Table 1.

The parameters required for the simulation is listed in Table 2. All parameters for simulation have been adjusted to observational data. The vehicles queue length based on numerical simulation is calculated by plotting vehicles density which was solved by Godunov method. Then, the vehicles queue length is obtained by summing up all cells which has zero speed or maximum density. At first, due to the availability of observational data, the simulation is only conducted for one cycle that is one period of red and green light signal.



Fig. 1: Video snapshot of vehicles queue at signalized intersection between Terusan Buah Batu street and Exit Toll Gate of Buah Batu taken from a drone.

The error of vehicles queue length by using LWR model is then calculated by comparing to observational data. Here, mean absolute percentage error (MAPE) is adopted. The MAPE is given by

$$MAPE = \frac{100}{n} \sum_{k=1}^n \left| \frac{L_o - L_s}{L_o} \right|$$

where n is the number of data, Lo and Ls are vehicles queue length of observation and simulation, respectively.

Table 1: Duration of traffic light signals and vehicles queue length.

Road section	Red signal	Green signal	Vehicles queue (observation)	Vehicles queue (theory)
Bojongsoang–Buah Batu	60 seconds	35 seconds	60.5 meters	61 meters
Buah Batu– Bojongsoang	60 seconds	35 seconds	60.5 meters	53.4 meters
Exit Tol	70 seconds	25 seconds	38.5 meters	38.2 meters

Fig. 2 depicts the average density of vehicles at at t = 60 seconds (red signal) and t = 70 seconds (green signal) for Bojongsoang – Buah Batu section. At initial, there is single car within 40 meters so that  $\rho_0 = 0.025$ . The position of single car is exactly in front of the traffic light signal. When the time is increasing, the length of vehicles queue is also changed. According to observational data, the maximum density is  $\rho_{max} = 0.19$ . The value 0.19 means that there are 19 cars within 100 meters.

Table 2: Parameters of each road for the simulation using LWR model.

Parameter	Bojongsoang–Buah Batu	Buah Batu–Bojongsoang	Buah Batu Toll Gate
Domain (m)	132	132	80
Cell size (m)	0.4	0.4	0.4
Time step (s)	0.01	0.01	0.01
Traffic light position (m)	66	66	40
Maximum speed (m/s)	6.94	6.11	6.94
Initial density (1/m)	0.025	0.025	0.025
Density at left hand boundary (1/m)	0.025	0.025	0.025
Maximum density (1/m)	0.19	0.19	0.19

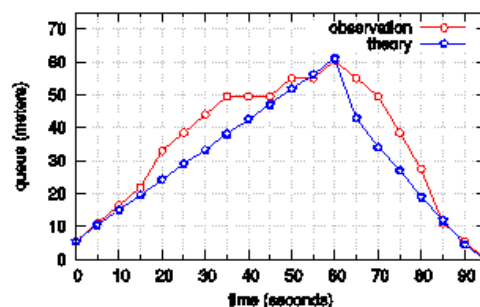


Fig. 2: Simulation result for average traffic density at t = 60 seconds (red signal) and t = 70 seconds (green signal) using LWR model.

Comparison of the vehicles queue length between observational data and simulation results for Bojongsong – Buah Batu section is shown by Fig. 3. Note that in the simulation, the number of arrival vehicles per unit time is assumed to be constant. In contrast, in the real situation, the number of arrival vehicles per unit time is obviously not constant. From the figure, it can be seen that the length of vehicles queue for the simulation (theory) during red signal period increases linearly while at green period, it is decreased almost linear. For this road section, it can be calculated that the MAPE is approximately 14.50%.

The length of vehicles queue against time during red and green signal period which is calculated by using the LWR model and the observation at Buah Batu – Bojongsong section is shown by Fig. 4. According to the figure, it can be seen that the fairly large error is shown during red signal period. It can also be observed from the figure that based on the observational data, from  $t = 10$  to  $t = 40$  seconds, there is no change in vehicles arrivals while from  $t = 40$  to  $t = 60$  seconds, the number of vehicles coming is changed significantly. In contrast, based on LWR theory, the number of vehicles is steadily increased. As consequence, the MAPE for this road section is about 20%.

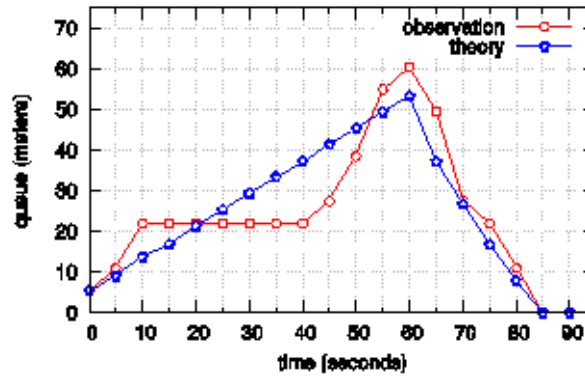


Fig. 3: Vehicles queue length calculated by LWR model (theory) and observational data at Bojongsong – Buah Batu section.

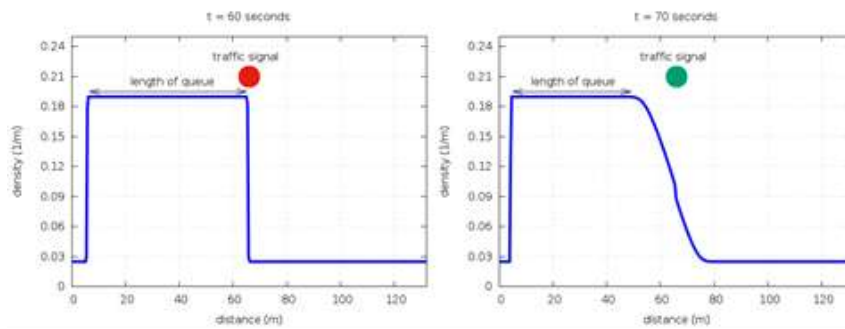


Fig. 4: Vehicles queue length calculated by LWR model (theory) and observational data at Buah Batu – Bojongsong section.

Fig. 5 reveals the vehicles queue length against time at Exit Toll Gate of Buah Batu with 70 seconds and 25 seconds of red and green signals period, respectively. It can be observed that the LWR model produces quite small error for this road section. Although, there is no change of the number of vehicles from  $t = 30$  to  $t = 70$  seconds, the resulted MAPE is about 8.52%. This error is better than other roads due to the vehicles arrival is almost linear.

According to three road sections being studied, it can be seen that the average accuracy of the LWR model is about 85.33%. Moreover, from Table 1, it can be calculated that the resulting accuracy at the end of red period for Bojongsong – Buah Batu section, Buah Batu – Bojongsong section, and Exit Toll Gate section is 99.17%, 88.26% and 99.22%, respectively.

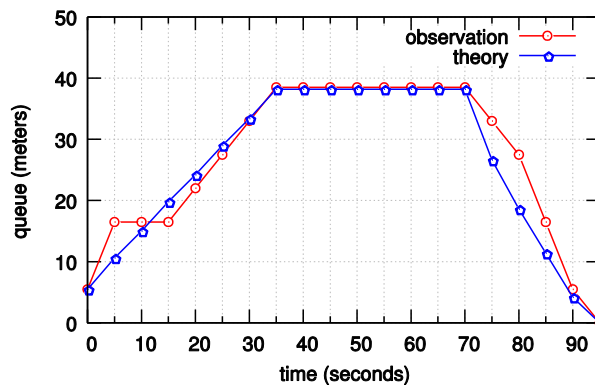
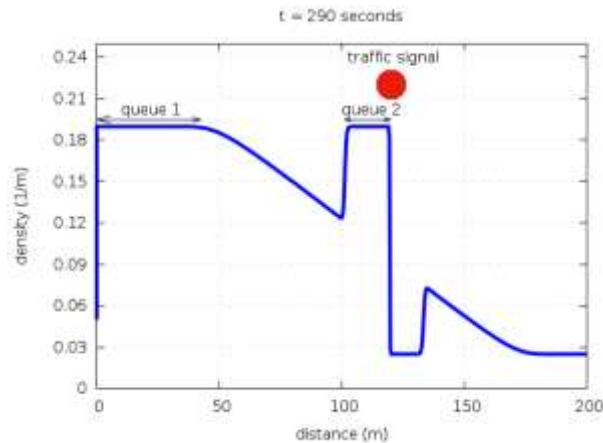


Fig. 5: Vehicles queue length calculated by LWR model (theory) and observational data at Exit Toll Gate of Buah Batu.



**Fig. 6:** Numerical result of vehicles queue at  $t = 290$  seconds for Buah Batu – Bojongsong road with density of left hand boundary  $0.05$   $1/m$ .

After the model is validated by observational traffic data then it can be used to predict the length of vehicles queue if the density from the left hand boundary is doubled. Fig. 6 displays density of vehicles in the fourth cycle at  $t = 290$  seconds or 5 seconds after green signal terminated. It can be seen that the traffic flow produces two queues. The length of the first and the second queue are  $28.6$  m and  $14$  m respectively. Further, at  $t = 313$  seconds, the two queues form one queue with the length of queue is  $120$  m. It can be observed that by doubling the density at the left hand boundary, the length of vehicles queue is also double.

## 5. Conclusion

Numerical simulation of vehicles queue in a signalized intersection has been successfully conducted. The numerical results have been validated by observational traffic data. It has been shown that LWR model which is solved by Godunov method yields accuracy about  $85.33\%$ . The growth of vehicles queue length by using LWR model is almost linear. It was calculated that the resulting accuracy at the end of red signal period for Bojongsong – Buah Batu section, Buah Batu – Bojongsong section, and Exit Toll Gate of Buah Batu section is  $99.17\%$ ,  $88.26\%$  and  $99.22\%$ , respectively. The numerical simulation for 4 cycles is also conducted. It has been observed that by doubling the density at the left hand boundary, the length of vehicles queue is also double. Suggestion for the future research is to use stochastic model to predict incoming vehicles so that it can obtain more accurate results.

## Acknowledgements

We acknowledge financial support from HIBAH BERSAING RISTEKDIKTI, Grant No. 2435/K4/KM/2016.

## References

- [1] Arara, A., Abousetta, E., & Drebbi, M. (2015). Simulation of Waiting Queues and Delay Distribution in Traffic Signals. *International Journal of Computer Science and Electronics Engineering*, 3(4), 331-334.
- [2] Gradinescu, V., Gorgorin, C., Diaconescu, R., Cristea, V., & Iftode, L. (2007, April). Adaptive traffic lights using car-to-car communication. In *2007 IEEE 65th Vehicular Technology Conference-VTC2007-Spring* (pp. 21-25). IEEE.
- [3] Guinot, V. (2003). *Godunov-Type Schemes: An Introduction for Engineers*. Elsevier.
- [4] Haberman, R. (1998). *Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic flow* (Vol. 21). SIAM.
- [5] LeVeque, R. J. (2002). *Finite Volume Methods for Hyperbolic Problems* (Vol. 31). Cambridge university press.
- [6] Lin, W. H., & Wang, C. (2004). An enhanced 0-1 mixed-integer LP formulation for traffic signal control. *IEEE Transactions on Intelligent transportation systems*, 5(4), 238-245.
- [7] Liu, H. X., Wu, X., Ma, W., & Hu, H. (2009). Real-time queue length estimation for congested signalized intersections. *Transportation research part C: emerging technologies*, 17(4), 412-427.
- [8] Lo, H. K. (1999). A novel traffic signal control formulation. *Transportation Research Part A: Policy and Practice*, 33(6), 433-448.
- [9] Lo, H. K., Chang, E., & Chan, Y. C. (2001). Dynamic network traffic control. *Transportation Research Part A: Policy and Practice*, 35(8), 721-744.
- [10] Mirchandani, P., & Head, L. (2001). A real-time traffic signal control system: architecture, algorithms, and analysis. *Transportation Research Part C: Emerging Technologies*, 9(6), 415-432.
- [11] Tarwidi, D., & Pudjaprasetya, S. R. (2013). Godunov method for Stefan problems with enthalpy formulations. *East Asian Journal on Applied Mathematics*, 3(02), 107-119.
- [12] Tarwidi, D. (2015, May). Godunov method for computerized lung cancer cryosurgery planning with efficient freezing time. In *Information and Communication Technology (ICoICT), 2015 3rd International Conference on* (pp. 494-499). IEEE.
- [13] Toro, E. F. (2013). *Riemann solvers and numerical methods for fluid dynamics: a practical introduction*. Springer Science & Business Media.
- [14] Treiber, M., & Kesting, A. (2013). *Traffic Flow Dynamics: Data, Models and Simulation*. Springer.
- [15] Wen, W. (2008). A dynamic and automatic traffic light control expert system for solving the road congestion problem. *Expert Systems with Applications*, 34(4), 2370-2381.
- [16] Wong, G. C. K., & Wong, S. C. (2002). A multi-class traffic flow model—an extension of LWR model with heterogeneous drivers. *Transportation Research Part A: Policy and Practice*, 36(9), 827-841.
- [17] van Leeuwen, J. S. (2006). Delay analysis for the fixed-cycle traffic-light queue. *Transportation Science*, 40(2), 189-199.
- [18] Yang, H., & Yagar, S. (1995). Traffic assignment and signal control in saturated road networks. *Transportation Research Part A: Policy and Practice*, 29(2), 125-139.
- [19] Yuan, S., Zhao, X., & An, Y. (2014). Identification and optimization of traffic bottleneck with signal timing. *Journal of Traffic and Transportation Engineering (English Edition)*, 1(5), 353-361.