Optimization of Multiprobe Placement for Computerized Cryosurgery Planning using Force-field Analogy

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***Abstract*—Cryosurgery is a medical technique to destroy unwanted tissue using extreme low temperature from a medium conducted using one or some cryoprobes with an minimum risk. Conventionally the configuration of cryoprobes (the number of cryoprobes and how to place them) has been applied using experience from trial and error methods of the cryosurgeon, so that it requires a method to optimize the configuration—to maximize the destruction of a target region while minimizing the destruction of surrounding healthy tissues. In this study, the heat transfer occurred in cryosurgery process is categorized as Stefan problem and simulated using finite volume scheme. Cryoprobe placement is optimized using deterministic displacement procedure based on the notion of force-field analogy. The optimization algorithm is evaluated to test some simple cases and complex-shaped geometry. The test cases are evaluated using perfect circle target with 4 different initial placements. Moreover, the algorithm is applied to the lung cancer geometry with the initial placement is obtained from bubble packing method. All different configurations are conducted using 4 to 14 cryoprobes. The simulation results show that the algorithm yields total defect average of 2.5% for circular geometry test cases and 3.68% for lung cancer with complex-shaped.**

***Keywords*—Optimization; Multiprobe placement; cryosurgery; enthalpy formulation; force-field analogy; simulation**

I. Introduction

Cryosurgery which is a technique to destroy unwanted tissues using extreme low temperature has been known to effectively destroy cancer with minimum cost and high healing rate. In 1961, the cryosurgery device has been developed from nitrogen cooling with only single cryoprobe to Joule-Thompson effect using 4 to 14 cryoprobes [1] to achieve better ways of conducting heat. Since it was developed, the cryorobes are placed using trial and error methods and the experience of the cryosurgeon. These methods will yield unwanted result such as unfrozen target tumor and damaged the surrounding healthy tissues. A successful cryosurgery is determined by minimum total defect or maximizing frozen target area with minimally damaged healthy tissues, so that the tumor tissues will not regrow. To achieve the objective, it needs an algorithm to place the cryoprobes properly. The number of cryoprobe used is also important since sometimes by using more cryoprobes are not necessarily more effective.

Many researchers has been done to achieve optimal placement, one of the techniques is force-field analogy that optimizes the cryoprobe placement with low computational cost. The algorithm tracks the total defect until the amount of unfrozen target (internal defect) is less than healthy tissue damaged (external defect). Then the total defect will be used in weight function to displace cryoprobes toward local optimum. The notion to optimize cryoprobe placement was proposed by Lung et al. [1] in a study to find such placement that fulfills the coverage of the frozen target area in prostate tumor case. Their results show that the algorithm is more efficient than other conventional optimization algorithms with total defect average of 2.98%. Rossi et al. [2], [3] also studied optimal cryoprobe placement in prostate tumor using bubble packing method and force-field analogy. The study showed that the frozen boundary obtained from numerical results differs 0.8 mm from experimental data. Other optimization approaches in multiprobe cryosurgery planning can be found in [4]–[9].

To the best of our knowledge, there is no research that applied optimization of multi-cryoprobe placement based on force-field analogy to lung cancer target. In this study, we use force-field analogy algorithm to optimize pre-initialized cryoprobe placement. Here, the numerical simulation of heat transfer in biological tissue is obtained by enthalpy method [10], [11] and solve it using finite volume scheme [12]. At first, we test the algorithm for simple circular-shaped target. There are 4 initial configurations that used in the test cases. For the computation, 4 to 14 cryoprobe is used for each configuration so that there are 44 test scenarios in total. Every scenario is evaluated to obtain minimum total defect. The algorithm is then applied to a complex-shaped geometry of lung cancer.

II. Computational Methods

A. Enthalpy Formulation

Stefan problem is a mathematical problem which discusses heat transfer in phase-change medium (solid and liquid) with moving interface between two phases [13]–[15]. We consider an area which consists of solid region (Ω_S) and liquid region (Ω_L). Fig. 1 illustrates two-dimensional Stefan problem with moving boundary Σ. The fix boundary on solid and liquid are $\partial_I\Omega$ and $\partial_{II}\Omega$ respectively.

The mathematical model of the Stefan problem consists of heat conduction equation in solid and liquid region with moving boundary between them. More detail about the model can be seen in [16]. Here, we adopt enthalpy method to solve

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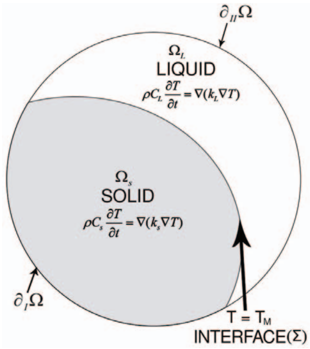

Fig. 1. Illustration of two-dimensional Stefan problem which consists of solid and liquid region.

such Stefan problem. By this method, the moving boundary between the solid and liquid phase are no longer taken into account so that the numerical scheme such as finite volume method is easily applied.

In enthalpy method, at first, we reformulate the temperature term in heat conduction equations into a single enthalpy equation. Let $E(\mathbf{x},t)$ be the enthalpy per unit area at position $\mathbf{x}$ and time t. The sum of sensible heat and latent can be formulated as

$$E(\mathbf{x},t) = \begin{cases} \rho c_S \left(T(\mathbf{x},t) - T_m\right), & T(\mathbf{x},t) < T_m, \\ \rho c_L \left(T(\mathbf{x},t) - T_m\right) + \rho L, & T(\mathbf{x},t) > T_m, \end{cases} \tag{1}$$

where $T(\mathbf{x},t) < T_m$ and $T(\mathbf{x},t) > T_m$ are temperature at solid and liquid phase respectively. Here, ρ, c_S, c_L, L, and T_m are density of biological tissue, specific heat of frozen tissue, specific heat of unfrozen tissue, latent heat, and melting point respectively.

B. *Finite Volume Discetization*

Suppose the two-dimensional domain of biological tissue is $0 \le x \le l_1$, $0 \le y \le l_2$. Then, the domain $[0, l_1]$ and $[0, l_2]$ are divided into M_1 and M_2 subintervals respectively. Thus, there are $M_1 M_2$ control volumes. Area inside $V_{i,j} = [x_{i-1/2}, x_{i+1/2}] \times [y_{j-1/2}, y_{j+1/2}]$ is called as control volume where $x_{i-1/2}$ is a node between x_{i-1} and x_i. The conservation of energy in each control volume $V_{i,j}$ can be expressed as

$$\int_{V_{i,j}} \left[E(\mathbf{x}, t+\Delta t) - E(\mathbf{x},t)\right] \mathrm{d}A = \int_t^{t+\Delta t} \int_{\partial V_{i,j}} -\mathbf{q}\cdot\mathbf{n}\, \mathrm{d}S\, \mathrm{d}t, \tag{2}$$

where $E(\mathbf{x},t)$ is the enthalpy per unit area, and $-\mathbf{q}\cdot\mathbf{n}$ is heat flux into the area $V_{i,j}$ across its boundary $\partial V_{i,j}$, $\mathbf{n}$ being the outgoing unit normal to $\partial V_{i,j}$ [12].

Finite volume discretization of (2) can be written as

$$\begin{aligned} E_{i,j}^{n+1} &= E_{i,j}^{n} + \frac{\Delta t}{\Delta x}\left[q_{i-1/2,j}^{n} - q_{i+1/2,j}^{n}\right] \\ &+ \frac{\Delta t}{\Delta y}\left[q_{i,j-1/2}^{n} - q_{i,j+1/2}^{n}\right] \end{aligned} \tag{3}$$

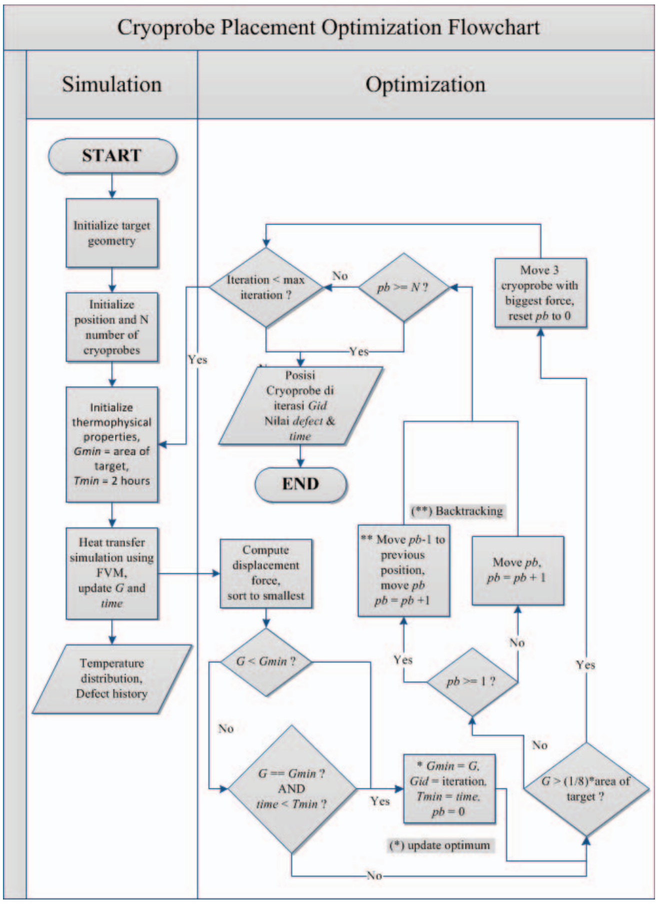

Fig. 2. The algorithm of optimization of multiprobe placement for computerized cryosurgery planning using force-field analogy and finite volume simulation.

where

$$q_{i-1/2,j} = \frac{T_{i-1,j} - T_{i,j}}{R_{i-1/2,j}},\; R_{i-1/2,j} = \frac{\Delta x}{2}\left(\frac{1}{k_{i-1,j}} + \frac{1}{k_{i,j}}\right),$$

$$q_{i,j-1/2} = \frac{T_{i,j-1} - T_{i,j}}{R_{i,j-1/2}},\; R_{i,j-1/2} = \frac{\Delta y}{2}\left(\frac{1}{k_{i,j-1}} + \frac{1}{k_{i,j}}\right).$$

Temperature distribution in biological tissue is obtained from (1) and (3):

$$T_{i,j} = \begin{cases} T_m + \dfrac{E_{i,j}}{\rho c_S}, & E_{i,j}^{n+1} \le 0 \quad \text{(solid)} \\ T_m, & 0 < E_{i,j} < \rho L \quad \text{(interface)} \\ T_m + \dfrac{E_{i,j} - \rho L}{\rho c_L}, & E_{i,j} \ge \rho L \quad \text{(liquid).} \end{cases} \tag{4}$$

III. Objective Function and Optimization

Since the objective of successful cryosurgery is to maximize target destruction and to minimize injured healthy tissue, unwanted result as named total defect will be considered throughout simulation and determined by the temperature in every control volume. The temperature threshold used to determine the defect is $-22\,°\mathrm{C}$ [1], which means that any control volume inside target area that has not reached below

TABLE I. PHYSICAL PROPERTIES OF HEALTY AND TUMOR TISSUE.

symbol	parameter	value	unit
c_s	specific heat of solid	1.7	kJ/kg/K
c_l	specific heat of liquid	4.1868	kJ/kg/K
k_s	thermal conductivity of solid	$2.66 \cdot 10^{-3}$	kJ/m/s/K
k_l	thermal conductivity of liquid	$0.6 \cdot 10^{-3}$	kJ/m/s/K
T_m	freezing point	273	K
L	latent heat	333.73	kJ/kg
ρ	density	1000	kg/m^3

threshold will be counted as (internal) defect and any control volume outside target area that has reached such value will be counted as (external) defect as well. Both values summed to gain insight of how simulation is regarded. The objective function for this problem can be formulated as:

$$G = \int_A w\, dA = \sum_k w_k A_k, \tag{5}$$

where G is called as defect function, A is the total area of the domain under consideration, w is a defect weighting function, and k is an index of control volume. Therefore, our goal is to minimize the defect function G with the defect weighting function is given by

$$w = \begin{cases} 1, & -22\,^{\circ}\mathrm{C} < T_k \quad \text{interior to the target region} \\ 0, & T_k \leq -22\,^{\circ}\mathrm{C} \quad \text{interior to the target region} \\ 1, & T_k \leq -22\,^{\circ}\mathrm{C} \quad \text{exterior to the target region} \\ 0, & -22\,^{\circ}\mathrm{C} < T_k \quad \text{exterior to the target region} \end{cases} \tag{6}$$

where T_k is temperature at control volume k.

To achieve our goal in minimizing the defect function, we adopt force-field analogy [1] to move the cryoprobes toward optimal position. Cryoprobe displacement using force-field analogy can be formulated as

$$\vec{F}_{nT} = \sum_k \frac{C_1}{|\vec{r}_{nk}|^2} w_k A_k \Delta T_k \vec{u}_{kn} \tag{7}$$

where $\vec{F}_{nT}$ is the force applied to cryoprobe n by all defects, C_1 is a constant, w_k is the weight function, A_k is the area of control volume k, $\vec{r}_{nk}$ is vector from control volume k to cryoprobe n, ΔT_k is the difference between temperature at index k and the temperature threshold, $\vec{u}_{nk}$ is a normalized version of $\vec{r}_{nk}$. To avoid cryoprobes stacking in a same place, a force is applied among cryoprobes and it is formulated as

$$\vec{F}_{nP} = \sum_j \frac{C_2}{|\vec{r}_{jn}|^3} \vec{u}_{jn} \tag{8}$$

where $\vec{F}_{nP}$ is the force applied to a cryoprobe by other cryoprobes, $\vec{r}_{jn}$ is vector position from a cryoprobe j to cryoprobe n. In this simulation, the values of $C_1 = 2 \times 10^{-4}$ and $C_2 = 1.2 \times 10^{-10}$. The displacement of a cryoprobe is total of both force, limited to 3 grid movements in any direction and maximum 3 simultaneous displacement in an iteration. The movement of cryoprobes follows:

$$\Delta \vec{r_n} = C_0 \left(\vec{F}_{nT} + \vec{F}_{np} \right) \tag{9}$$

where $\Delta \vec{r_n}$ is the displacement of cryoprobes while C_0 is a constant.

The algorithm of optimization of multiprobe placement for computerized cryosurgery planning using force-field analogy is shown in the flow chart in Fig. 2. At first, 3 cryoprobes with greatest force will be displaced in every iteration until the total defect has reached 1/8 of target area, then only a cryoprobe with greatest force will be displaced. In the latter part of optimization process, if a displacement has increasing total defect, backtracking algorithm will try to displace next greatest force cryoprobe one by one untill better solution is found. This ensures the solution is improved toward local optimum. The optimization continues until either it has reached 100 iterations or there is no cryoprobe displacement that has better solution.

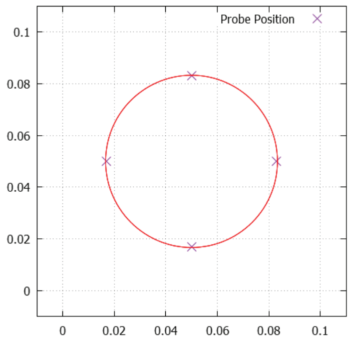

Fig. 3. Initial position of Config 1 which is started by 4 cryoprobes.

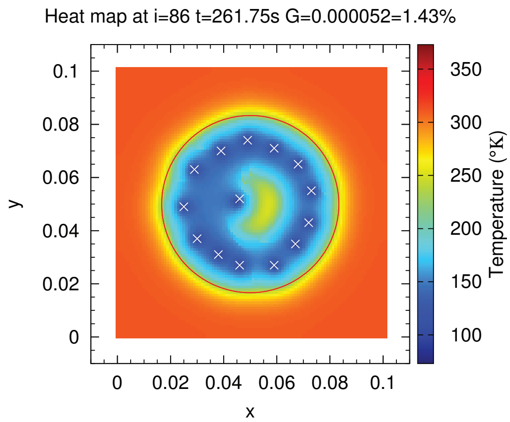

Fig. 4. Optimal solution of Config 1 with 14 cryoprobes and total defect about 1.43%.

IV. Results and Discussion

A. Test Cases

To evaluate our algorithm, we consider four test cases namely Config 1, Config 2, Config 3, and Config 4. For all cases, we propose circular target region with different initial placements of cryoprobes. Here, 4 to 14 cryoprobes are used in each configuration so that there are 44 scenarios that should be tested. All parameters of this simulation are summarized in Table I.

In Config 1, four cryoprobes is initially placed on the boundary of the target, see Fig. 3. Numerical results using force-field analogy to obtain optimal placement of cryoprobes are summarized in Table II. From that table, we can see that configuration with 14 cryoprobes has the smallest total defect approximately 1.43% with elapsed time about 261 seconds.

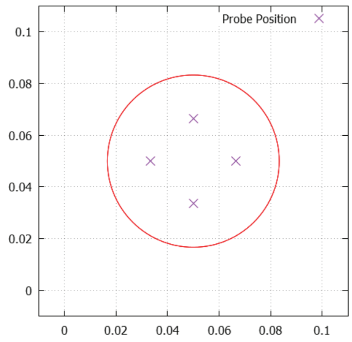

Fig. 5. Initial position of Config 2 which is started by 4 cryoprobes.

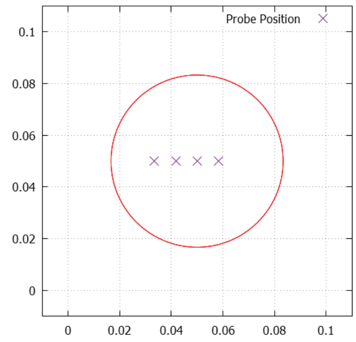

Fig. 7. Initial position of Config 3 which is started by 4 cryoprobes.

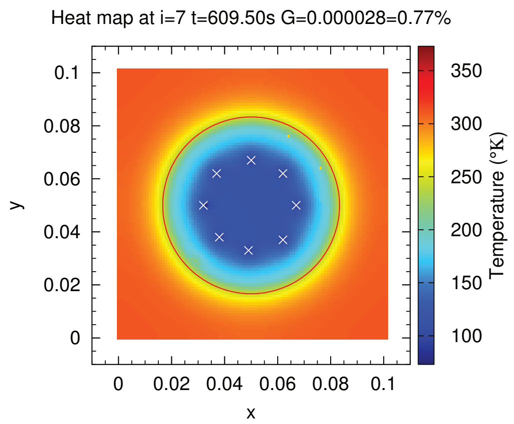

Fig. 6. Optimal solution of Config 2 with 8 cryoprobes and total defect about 0.77%.

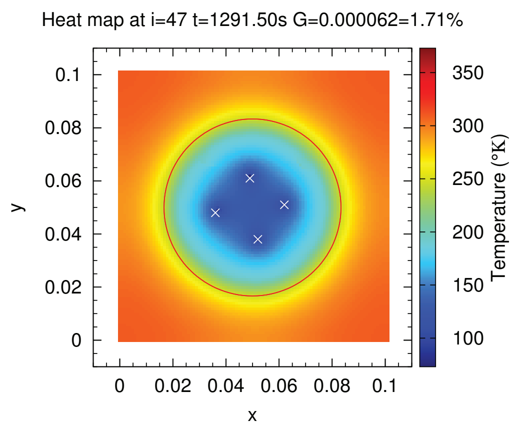

Fig. 8. Optimal solution of Config 3 with 4 cryoprobes and total defect about 1.71%.

Fig. 4 presents the optimal solution of Config 1 with 14 cryoprobes. The simulation results show that the cryoprobes move inward and the shaped looks like the target region itself. For all cryoprobes in this configuration, we have total defect average about 2.62%.

In Config 2, the cryoprobes position is initially placed on half of the radius of target region which is illustrated in Fig. 5. Numerical results of this configuration are listed in Table III. From that table, we can see that configuration with 8 cryoprobes has the smallest total defect approximately 0.77%. Fig 6 displays optimal solution of Config 2 with 8 cryoprobes. For all cryoprobes in this case, it has total defect average of 1.30%.

Table IV summarizes numerical results of Config 3 where the initial position of cryoprobes is on the horizontal line at the middle of circle (Fig. 7). This configuration has slightly worse results than the others with total defect average about 3.00%. The optimal solution of Config 3 is shown in Fig 8. Furthermore, in Config 4, the initial position of cryoprobes is similar to Config 1 but there is one cryoprobe at the center of circle, see Fig. 9. Numerical results for Config 4 are listed in Table V. From that table, we can see that the configuration with 13 cryoprobes has the smallest total defect approximately 1.68% with elapsed time about 250 seconds. Fig. 10 depicts the optimal solution of Config 4 with 13 cryoprobes. In this configuration, the average of total defect is about 3.08%.

TABLE II. NUMERICAL RESULTS OF CONFIG 1.

#probes	Optimum found at iteration	#iteration	total defect (%)	elapsed time (s)
4	41	45	3.10	1168.75
5	33	38	3.39	853.5
6	35	41	4.03	658.25
7	24	31	3.23	541
8	28	36	2.81	467.5
9	49	58	1.77	422.25
10	63	73	2.57	377.25
11	59	70	1.82	328.5
12	97	100	2.34	316.25
13	60	73	2.34	307.25
14	**86**	**100**	**1.43**	**261.75**

TABLE III. NUMERICAL RESULTS OF CONFIG 2.

#probes	Optimum found at iteration	#iteration	total defect (%)	elapsed time (s)
4	11	15	2.92	1169.25
5	4	9	2.15	898.25
6	5	11	1.38	747.75
7	5	12	1.27	654
8	**7**	**15**	**0.77**	**609.75**
9	21	30	0.86	546.25
10	17	27	1.10	518.75
11	22	33	1.05	489.75
12	13	25	1.08	464.25
13	27	40	0.83	447.5
14	45	59	0.91	414.25

It is also noted that the speed of obtaining the optimal solution is based on how different the initial placements

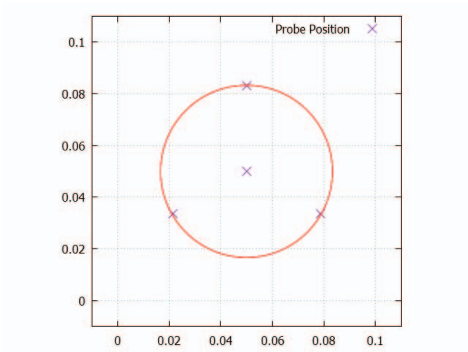

Fig. 9. Initial position of Config 4 which is started by 4 cryoprobes.

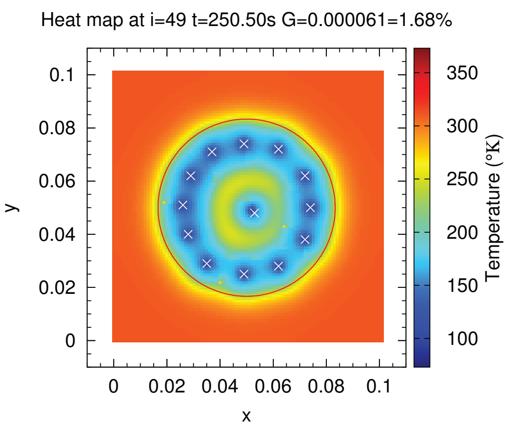

Fig. 10. Optimal solution of Config 4 with 13 cryoprobes and total defect about 1.68%.

TABLE IV. NUMERICAL RESULTS OF CONFIG 3.

#probes	Optimum found at iteration	#iteration	total defect (%)	elapsed time (s)
4	**47**	**51**	**1.71**	**1291.5**
5	85	90	2.26	1086.25
6	99	100	2.54	940
7	99	100	2.48	916.5
8	99	100	2.70	804.25
9	95	100	2.54	722.75
10	94	100	3.59	676.75
11	94	100	3.03	655.25
12	98	100	3.70	630
13	98	100	4.30	596.25
14	52	66	4.19	579.25

TABLE V. NUMERICAL RESULTS OF CONFIG 4.

#probes	Optimum found at iteration	#iteration	total defect (%)	elapsed time (s)
4	22	26	5.13	1313.25
5	22	27	4.08	965.25
6	43	49	3.14	750.5
7	43	50	3.14	597.75
8	79	87	2.48	510
9	38	47	3.70	405.25
10	15	25	3.78	325.25
11	43	54	2.48	297.75
12	65	77	1.82	282
13	**49**	**62**	**1.68**	**250.5**
14	38	52	2.48	235.25

in each configuration. Fig 11 illustrates total defect for all configurations against number of cryorpobes. In this figure,

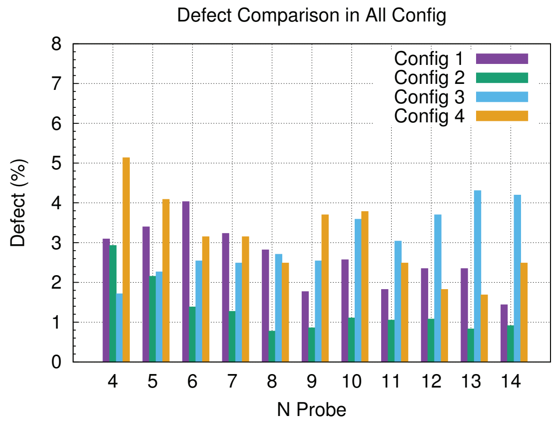

Fig. 11. Comparison of total defect for all configurations against number of cryorpobes.

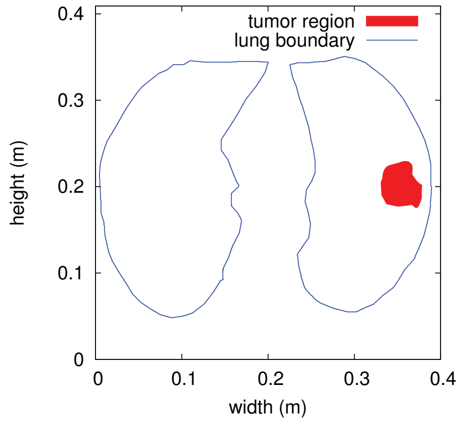

Fig. 12. Illustration of complex-shaped target geometry of lung cancer.

we can see that Config 2 with 8 cryoprobes has the minimum of total defect about 0.77%. The figure also reveals that for all configurations the total defect average is about 2.5% with maximum of 5.13% and minimum of 0.77%.

B. Complex-shaped Geometry

The algorithm is then applied in complex-shaped target geometry. The target region is lung cancer which is illustrated in Fig. 12. Here, the initial configuration of force-field analogy optimization is obtained from bubble packing method which has been calculated by [17]. The initial cryoprobes position using 9 cryprobes obtained from bubble packing method is shown in Fig 13. This configuration is used to prove the reliability of force-field analogy algorithm in complex-shaped geometry.

At first, the initial placement has total defect of 7.27% which is not yet optimal. The optimal solution using force-field analogy is obtained at iteration 31 with total defect of 3.68%. The optimal placement of cryoprobes is depicted in Fig 14. Although it has not been tried to use other number of cryoprobes, current result has already considered to be an effective and successful optimization because the total defect has been decreased by half.

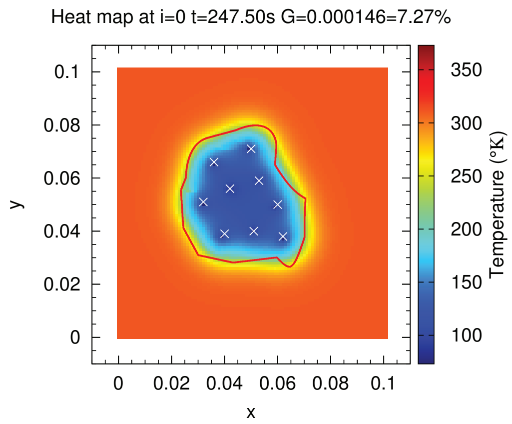

Fig. 13. Initial placement obtained from bubble packing method using 9 cryoprobes.

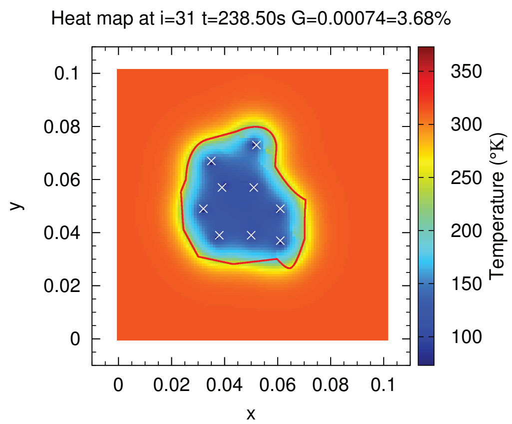

Fig. 14. Optimal placement using force-field analogy for lung cancer cryosurgery with complex-shaped geometry.

V. Conclusion and Future Works

The optimal results in all configurations that were evaluated using circular target with 4 initial placements and using 4 to 14 cryoprobes have total defect average about 2.50% with maximum of 5.13% and minimum of 0.77%. The numerical results shows that the cryoprobe placement algorithm using force-field analogy has successfully found the optimal solutions. The complex-shaped geometry with the initial placement obtained from bubble packing method has the optimal solution with total defect of 3.68%. It can be said that the optimization algorithm has successfully found a better solution from the initial placement. Hence it can be concluded that optimization of cryoprobe placement in cryosurgery planning using force-field analogy is an effective way to find an optimal cryoprobe placement. Suggestion for future research is to develop three-dimensional simulation to obtain more accurate results.

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