Fakultas Informatika
School of Computing
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CNH2B4 / KOMPUTASI NUMERIK

TIM DOSEN

KK MODELING AND COMPUTATIONAL EXPERIMENT

5 SOLUSI SISTEM PERSAMAAN LINEAR: METODE ITERASI JACOBI & GAUSS-SEIDEL

Iterasi Jacobi & Gauss-Seidel

Sistem persamaan linear:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n = b_3$$
$$\ldots$$
$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n$$

$a_{kk} \neq 0$, k = 0, 1, 2, 3, …, n

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)} \ldots - a_{1n}x_n^{(k)}}{a_{11}}$$
$$x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k)} - a_{23}x_3^{(k)} \ldots - a_{2n}x_n^{(k)}}{a_{22}}$$
$$x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k)} - a_{32}x_2^{(k)} \ldots - a_{3n}x_n^{(k)}}{a_{33}}$$
$$\ldots$$
$$x_n^{(k+1)} = \frac{b_n - a_{n1}x_1^{(k)} - a_{n2}x_2^{(k)} \ldots - a_{nn-1}x_{n-1}^{(k)}}{a_{nn}}$$

Iterasi Jacobi

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)} \ldots - a_{1n}x_n^{(k)}}{a_{11}}$$
$$x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)} \ldots - a_{2n}x_n^{(k)}}{a_{22}}$$
$$x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)} \ldots - a_{3n}x_n^{(k)}}{a_{33}}$$
$$\ldots$$
$$x_n^{(k+1)} = \frac{b_n - a_{n1}x_1^{(k+1)} - a_{n2}x_2^{(k+1)} \ldots - a_{nn-1}x_{n-1}^{(k+1)}}{a_{nn}}$$

Iterasi Gauss-Seidel

Contoh

- Selesaikan SPL berikut dengan menggunakan Iterasi Jacobi dan Iterasi Gauss-Seidel dengan nilai awal $(x_1,x_2,x_3,x_4) = (0,0,0,0)$.

$$10x_1 - x_2 + 2x_3 = 6$$
$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$
$$2x_1 - x_2 + 10x_3 - x_4 = -11$$
$$3x_2 - x_3 + 8x_4 = 15$$

Jacobi Iterative

To solve $A\mathbf{x} = \mathbf{b}$ given an initial approximation $\mathbf{x}^{(0)}$:

INPUT the number of equations and unknowns n; the entries a_{ij}, $1 \le i, j \le n$ of the matrix A; the entries b_i, $1 \le i \le n$ of $\mathbf{b}$; the entries XO_i, $1 \le i \le n$ of $\mathbf{XO} = \mathbf{x}^{(0)}$; tolerance TOL; maximum number of iterations N.

OUTPUT the approximate solution $x_1, \ldots, x_n$ or a message that the number of iterations was exceeded.

Step 1 Set $k = 1$.

Step 2 While ($k \le N$) do Steps 3–6.

Step 3 For $i = 1, \ldots, n$

$$\text{set } x_i = \frac{1}{a_{ii}}\left[-\sum_{\substack{j=1\\j\ne i}}^{n} (a_{ij}XO_j) + b_i\right].$$

Step 4 If $||\mathbf{x} - \mathbf{XO}|| < TOL$ then OUTPUT $(x_1, \ldots, x_n)$;
(*The procedure was successful.*)
STOP.

Step 5 Set $k = k + 1$.

Step 6 For $i = 1, \ldots, n$ set $XO_i = x_i$.

Step 7 OUTPUT ('Maximum number of iterations exceeded');
(*The procedure was successful.*)
STOP. ■

Burden, Richard L., and J. Douglas Faires. *Numerical Analysis*. Brooks/Cole, *USA*, 2001.

Gauss-Seidel Iterative

To solve $A\mathbf{x} = \mathbf{b}$ given an initial approximation $\mathbf{x}^{(0)}$:

INPUT the number of equations and unknowns n; the entries a_{ij}, $1 \leq i, j \leq n$ of the matrix A; the entries b_i, $1 \leq i \leq n$ of $\mathbf{b}$; the entries XO_i, $1 \leq i \leq n$ of $\mathbf{XO} = \mathbf{x}^{(0)}$; tolerance TOL; maximum number of iterations N.

OUTPUT the approximate solution $x_1, \ldots, x_n$ or a message that the number of iterations was exceeded.

Step 1 Set $k = 1$.

Step 2 While ($k \leq N$) do Steps 3–6.

Step 3 For $i = 1, \ldots, n$

$$\text{set } x_i = \frac{1}{a_{ii}}\left[-\sum_{j=1}^{i-1} a_{ij}x_j - \sum_{j=i+1}^{n} a_{ij}XO_j + b_i\right].$$

Step 4 If $||\mathbf{x} - \mathbf{XO}|| < TOL$ then OUTPUT $(x_1, \ldots, x_n)$;
(*The procedure was successful.*)
STOP.

Step 5 Set $k = k + 1$.

Step 6 For $i = 1, \ldots, n$ set $XO_i = x_i$.

Step 7 OUTPUT ('Maximum number of iterations exceeded');
(*The procedure was successful.*)
STOP.

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Iterasi Jacobi & Gauss-Seidel

- Kriteria konvergensi (penghentian iterasi) menggunakan L_∞-norm:

$$\frac{\left\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\right\|_\infty}{\left\|\mathbf{x}^{(k)}\right\|_\infty} < TOL$$

- dengan

$$\left\|\mathbf{x}\right\|_\infty = \max_{1 \le i \le n} \left|x_i\right|$$

- Syarat cukup agar iterasinya konvergen adalah sistem dominan secara diagonal

$$\left|a_{ii}\right| > \sum_{j=1,\, j \ne i}^{n} \left|a_{ij}\right|$$

Iterasi Jacobi & Gauss-Seidel

- Kriteria konvergensi (penghentian iterasi) dapat juga menggunakan L_2-norm:

$$\frac{\left\|\mathbf{x}^{(k)}-\mathbf{x}^{(k-1)}\right\|_2}{\left\|\mathbf{x}^{(k)}\right\|_2} < TOL$$

- dengan

$$\|\mathbf{x}\|_2 = \left\{\sum_{i=1}^{n} x_i^2\right\}^{1/2}$$

Latihan

- Selesaikan SPL berikut dengan menggunakan Iterasi Jacobi dan Iterasi Gauss-Seidel dengan nilai awal $(x_1,x_2,x_3) = (0,0,0)$ sampai 2 iterasi.

$$3x_1 - x_2 + x_3 = 1$$
$$3x_1 + 6x_2 + 2x_3 = 0$$
$$3x_1 + 3x_2 + 7x_3 = 4$$

Latihan

- Selesaikan SPL berikut dengan menggunakan Iterasi Jacobi dan Iterasi Gauss-Seidel dengan nilai awal $(x_1,x_2,x_3,x_4) = (0,0,0,0)$ sampai 2 iterasi.

$$
\begin{aligned}
4x_1 + x_2 - x_3 + x_4 &= -2 \\
x_1 + 4x_2 - x_3 - x_4 &= -1 \\
-x_1 - x_2 + 5x_3 + x_4 &= 0 \\
x_1 - x_2 + x_3 + 3x_4 &= 1
\end{aligned}
$$

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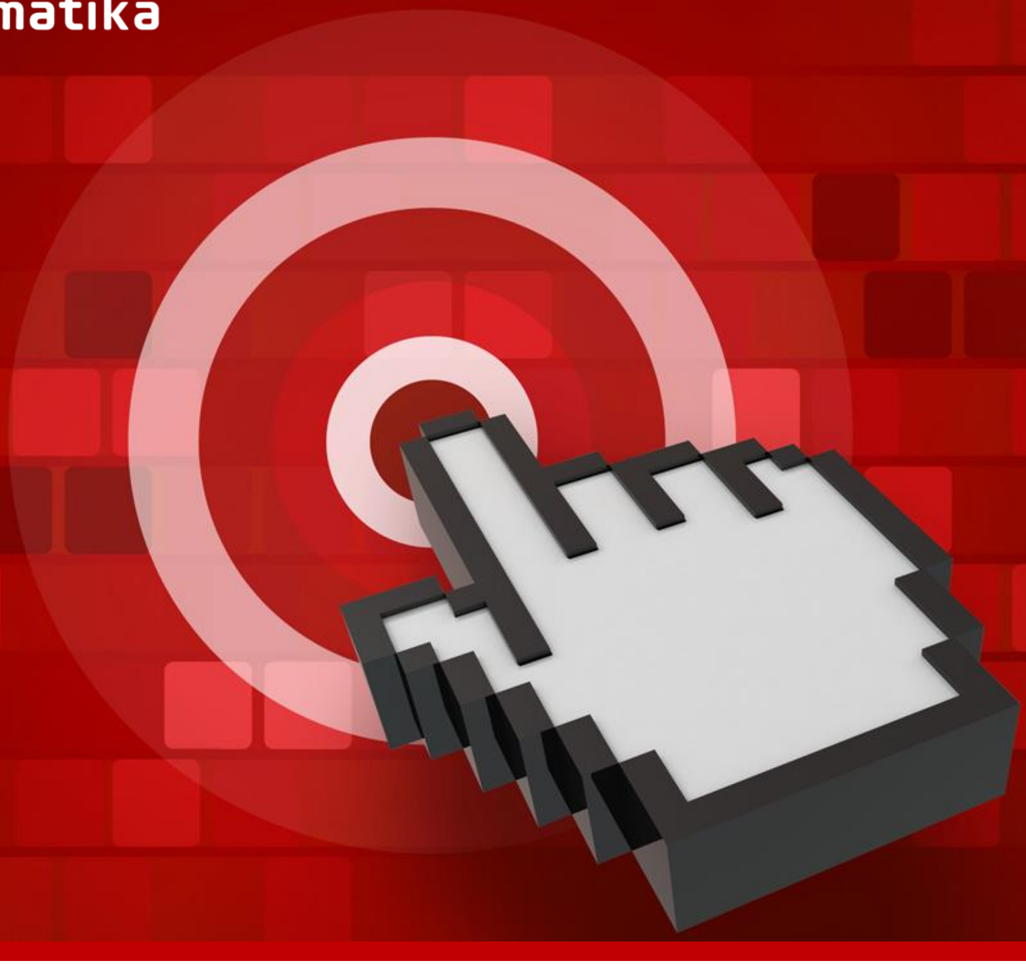

THANK YOU